

Algebraic Geometry Sheet 1

Problem 1. *Show that over a finite field, the evaluation map does not determine a polynomial uniquely.*

Problem 2. *Determine whether the following sets are algebraic:*

1. $\{(\cos t, \sin t); t \in [0, 2\pi]\} \subset \mathbb{A}_{\mathbb{R}}^2$
2. $\{(t, \sin t); t \in \mathbb{R}\} \subset \mathbb{A}_{\mathbb{R}}^2$

Problem 3. *For an ideal J in the ring R , prove that the radical*

$$\sqrt{J} := \{r \in R \mid \exists k \geq 1 \text{ with } r^k \in J\}$$

is also an ideal.

Problem 4. *Consider the following ideals in $\mathbb{Z}[x]$:*

$$I_1 = (x), \quad I_2 = (x^2 + 1).$$

1. *Prove that they are prime ideals.*
2. *Are they radical ideals?*
3. *Show that they are not maximal ideals.*

Problem 5. *Consider the following ideals in $\mathbb{C}[x, y, z]$:*

$$I_1 = (xy+y^2, xz+yz), I_2 = (xy+y^2, xz+yz+xyz+y^2z), I_3 = (xy^2+y^3, xz+yz).$$

For each $k, l = 1, 2, 3$ and $k < l$, determine:

1. *Is $I_k = I_l$?*
2. *Is $V(I_k) = V(I_l)$?*
3. *Is $I_k = I(V(I_k))$?*