

Algebraic Geometry Sheet 14

Problem 1. *Suppose that we are working over \mathbb{C} and let $X \subset \mathbb{P}^3$ be a quadric surface. Recalling that $X \cong \mathbb{P}^1 \times \mathbb{P}^1$, show that the lines in X are precisely those corresponding to lines of the form $\mathbb{P}^1 \times \{\text{pt}\}$ or $\{\text{pt}\} \times \mathbb{P}^1$ in $\mathbb{P}^1 \times \mathbb{P}^1$.*

Problem 2. *Suppose that k is an algebraically closed field and that $\text{char } k = 3$. Consider the cubic surface $X \subset \mathbb{P}_k^3$ given by the equation:*

$$x^2y + y^2z + z^2w + w^2x = 0.$$

Prove that X is smooth. How many lines are there in X ?

Problem 3. *Consider the Fermat surface $X \subset \mathbb{P}_k^3$ given by the equation:*

$$x^4 + y^4 + z^4 + w^4 = 0.$$

Prove that when $k = \mathbb{C}$ this surface contains 48 lines. Show that over the field \mathbb{F}_9 there are more.