Leibniz Universität Hannover

Institut für Algebraische Geometrie
Prof. Dr. M. Schütt,
Dr. N. Broomhead
Abgabe: 22.10.2009

Algebraic Geometry
Sheet 2

Unless otherwise stated, you should assume that we are working over an algebraically closed field k.

Problem 1. *Prove that the Zariski-topology on $\mathbb{A}^2_{\mathbb{C}}$ is not the same as the product topology on $\mathbb{A}^1_{\mathbb{C}} \times \mathbb{A}^1_{\mathbb{C}} = \mathbb{A}^2_{\mathbb{C}}$ as follows:*

1. *Determine all the Zariski closed subsets of $\mathbb{A}^1_{\mathbb{C}}$.*
2. *Determine all the closed subsets in the product topology on $\mathbb{A}^1_{\mathbb{C}} \times \mathbb{A}^1_{\mathbb{C}}$.*
3. *Give an example of a Zariski closed subset of $\mathbb{A}^2_{\mathbb{C}}$ which is not closed in the product topology.*

Problem 2. *Let $I = (x^2 - yz, xz - x) \subset \mathbb{C}[x, y, z]$ and $X = V(I)$. Determine the irreducible components of X (and prove their irreducibility).*

Problem 3. *Let $X = \{(t, t^2, t^3) \mid t \in k\} \subset \mathbb{A}^3$. Show that X is an affine variety (i.e. irreducible) of dimension 1 and compute $I(X)$.*

Problem 4. *Let $X \subset \mathbb{A}^2$ be an irreducible algebraic set. Show that either*

1. *$X = V(0)$, i.e. X is the whole space $\mathbb{A}^2$, or*
2. *$X = V(f)$ for some irreducible polynomial $f \in k[x, y]$, or*
3. *$X = V(x - a, y - b)$ for some $a, b \in k$, i.e. X is a single point.*

Deduce that $dim(\mathbb{A}^2) = 2$.
Hint: *Show that the common zero locus of two polynomials $f, g \in k[x, y]$ without a common factor is finite.*