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## Algebraic Geometry Sheet 3

Unless otherwise stated, you should assume that we are working over an algebraically closed field  $k$ .

**Problem 1.** Let  $X \in \mathbb{A}^3$  be the union of the three coordinate axes. Determine the ideal  $I(X)$  and show that it can't be generated by fewer than 3 elements, even though  $X$  has codimension 2 in  $\mathbb{A}^3$ .

**Problem 2.** An algebraic set  $X \in \mathbb{A}^2$  defined by a polynomial of degree 2 is called a conic. Show that the coordinate ring of any irreducible conic is isomorphic to the coordinate ring of either  $V(y - x^2)$  or  $V(xy - 1)$ .

**Problem 3.** Prove that  $\mathcal{O}_{\mathbb{C}^2}(\mathbb{C}^2 \setminus \{(0, 0)\}) = \mathcal{O}_{\mathbb{C}^2}(\mathbb{C}^2)$ .

**Problem 4.** Let  $X = \{x, y\}$  be the 2 point topological space, with the discrete topology (i.e. the open sets are  $\emptyset, \{x\}, \{y\}, X$ ). Construct an example of a pre-sheaf on  $X$  which is not a sheaf.