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Algebraic Geometry
Sheet 6

You should assume that we are working over an algebraically closed field.

Problem 1. *Let X be a prevariety. Prove carefully that any open subset of X is also a prevariety.*

Problem 2. *Let $(X, \mathcal{O}_X)$ be a prevariety and let $Y \subset X$ be an irreducible closed subset. For every open subset $U \subset Y$, define $\mathcal{O}_Y(U)$ to be the ring of k valued functions f on U with the following property: for every point $P \in Y$ there is a neighbourhood V of P in X and a section $F \in \mathcal{O}_X(V)$ such that f coincides with F on U.*

1. *Show that the rings $\mathcal{O}_Y(U)$ together with the obvious restriction maps define a sheaf $\mathcal{O}_Y$ on Y.*
2. *Show that $(Y, \mathcal{O}_Y)$ is a prevariety.*

Problem 3. *Prove that any rational map from a nonempty open subset $U \subset \mathbb{A}^1$ to $\mathbb{A}^1$,*

$$\varphi : U \to \mathbb{A}^1$$

can be extended uniquely to a morphism of prevarieties,

$$\widetilde{\varphi} : \mathbb{P}^1 \to \mathbb{P}^1.$$

Problem 4. *Prove that the prevariety $\mathbb{P}^1$ is a variety.*