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Algebraic Geometry
Sheet 7

You should assume that we are working over an algebraically closed field.

Problem 1. *Let $S = \bigoplus_{d=0}^{\infty} S^{(d)}$ be a graded ring.*

1. *Prove that a homogeneous ideal I is prime if and only if, for any* **homogeneous** *elements $f, g \in S$ such that $fg \in I$ then $f \in I$ or $g \in I$.*
2. *Let I, J be homogeneous ideals. Show that $I + J, IJ, I \cap J$ and $\sqrt{I}$ are homogeneous ideals.*

Problem 2. *Let P_1, P_2, P_3 and Q_1, Q_2, Q_3 be two triples of distinct points in $\mathbb{P}^1$. Prove that there exists an automorphism of $\mathbb{P}^1$ which maps $P_i \mapsto Q_i$ for all $i = 1, 2, 3$.*

Problem 3. *For $d \geq 1$ consider*

$$\begin{array}{rccc} \varphi_d : & \mathbb{P}^1 & \to & \mathbb{P}^d \\ & (x_0 : x_1) & \mapsto & (x_0^d : x_0^{d-1}x_1 : \cdots : x_1^d) \end{array}$$

Determine whether the image is a projective variety.

Problem 4. *Prove that the image of the map*

$$\begin{array}{rccc} \varphi : & \mathbb{P}^1 \times \mathbb{P}^1 & \to & \mathbb{P}^3 \\ & ((x_0 : x_1), (y_0 : y_1)) & \mapsto & (x_0y_0 : x_0y_1 : x_1y_0 : x_1y_1) \end{array}$$

can be given as $Z(a_0a_3 - a_1a_2)$. Furthermore, prove that this map gives a bijection from $\mathbb{P}^1 \times \mathbb{P}^1$ to $Z(a_0a_3 - a_1a_2)$.