Stefan Schreieder Summer term 2026

Working Seminar: Brauer Group

Tuesday 14:15-15:45 in F 107

This seminar discusses the Brauer group of a field and Grothendieck's generalization to schemes. We compare different definitions, discuss applications in algebraic geometry and discuss open problems, such as the period index conjecture.

The talks are naturally grouped into the following blocks:

1. Brauer group of a field (talks 1-3)
2. Brauer group of a scheme (talks 4-6)
3. Azumaya algebras, gerbes and twisted sheaves (talks 7,8)
4. period index conjecture (talks 9-12)

The program below contains a suggestion how the material of a given block can be distributed among the various talks. This is not necessarily binding, but I ask you to discuss changes with me. The participants of a given block should discuss the content of their talks in separate meetings.

If you would like to give a talk and join one of the four "sub-working groups", please write an email to schreieder@math.uni-hannover.de.

Some parts of the program are provisional and up for discussion.

1. **Distribution of talks** (DM)

   Date: 07.04.2026

2. **Brauer group of a field I: Quaternion algebras** (SF)

   Date: 14.4.

   Reading: [GS06, §1]

   Content: Discuss the example of quaternion algebras from [GS06, §1] in some detail.

3. **Brauer group of a field II: Wedderburn, Galois descent and cyclic algebras** (RL)

   Date: 21.4

   Reading: [GS06, §2.1-§2.5]

   Content: Sketch the proof of Wedderburns theorem [GS06, §2.1] and discuss the concept of splitting fields [GS06, §2.2]. Introduce the Brauer group [GS06, §2.4] and mention the example of cyclic algebras [GS06, §2.5].

4. **Brauer group of a field III: Cohomological interpretation and Severi–Brauer varieties** (JL)

   Date: 28.4.

   Reading: [GS06, §2.7, §4, §5]

Content: Explain the alternative interpretation of the Brauer group in terms of Galois cohomology from [GS06, §4.4, 4.5]. Introduce the notion of Severi–Brauer varieties and explain the relation to it, see [GS06, §5].

5. **Brauer groups of schemes I: Basics** (MK)

   Date: 5.5.

   Reading: [CTS21, §2, 3.1,3.2], [Mil80, §9]

   Content: Recall briefly the theory of étale cohomology, cf. [CTS21, §2] and [Mil80]. Introdue the Brauer-Grothendieck group of a scheme [CTS21, §3.2] and discuss basic properties.

6. **Brauer groups of schemes II: Purity, C_1-fields, Tsen's theorem** (AF)

   Date: 12.5.

   Reading: [CTS21, §3.4-3.8], [GS06, §6.2]

   Content: Discuss the relation to the Brauer group of the generic point [CTS21, §3.5] and purity [CTS21, §3.7]. Discuss C_1-fields, their Brauer groups [GS06, Proposition 6.2.3] and Tsen's theorem [GS06, Theorem 6.2.8] and conclude [CTS21, Thm 5.6.1(ii)].

7. **Brauer groups of schemes III: Applications to rationality** (NA)

   Date: 19.5.

   Reading: [CTS21, §6], [Bea16, §6]

   Content: Present Beauville's account on Brauer groups in [Bea16, §6.2] and his view on the Artin–Mumford example, see [Bea16, §6.3] and the references therein. For a more algebraic point of view and more examples of rationally connected varieties with nontrivial Brauer groups, see [CTS21, §6].

8. **Azumaya algebras, gerbes, and twisted sheaves, I** (SM)

   Date: 2.6.

   Reading: [CTS21, §3.1, §4], [Ols16, §12.2,12.3], [vB24, §3.1], [dJ]

   Content: This is the first talk of a series of two—please split the content suitably.

   The goal of this series is as follows: Introduce the Brauer–Azumaya group [CTS21, §3.1] and discuss the comparison to the cohomological/Grothendieck definition, see [CTS21, §3.3]. Introduce stacks, the language of gerbes, and twisted sheaves, see [CTS21, §4.1] and the references therein. Sketch de Jong's proof of Gabber's theorem, see [CTS21, §4.2] and [dJ].

9. **Azumaya algebras, gerbes, and twisted sheaves, II** (PF)

   Date: 9.6.

   Reading: [CTS21, §3.1, §4], [Ols16, §12.2,12.3], [vB24, §3.1], [dJ]

   Content: This is the second talk of a series of two—please split the content suitably.

   The goal of this series is as follows: Introduce the Brauer–Azumaya group [CTS21, §3.1] and discuss the comparison to the cohomological/Gorthendieck definition, see [CTS21, §3.3]. Introduce stacks, the language of gerbes, and twisted sheaves, see [CTS21, §4.1] and the references therein. Sketch de Jong's proof of Gabber's theorem, see [CTS21, §4.2] and [dJ].

10. **Period–index: From unramified to ramified classes** (LM)

    Date: 16.6.

Reading: [GS06, §2.8, §4.5], [Sta08, §4.6], [dJS10], [GH26]

Content: Discuss the period and index of Brauer classes and the basic relations from [GS06, §2.8, §4.5]. State the period-index conjecture, see e.g. [dJP22, Conjecture 1.1]. Discuss the results in [GH26] and explain how they allow to reduce the problem from the ramified to the unramified case, see [GH26, Theorem 4.1] and the original references [dJ04, dJS10].

11. **Period–index for surfaces** (LZ)

    Date: 23.6.

    Reading: [dJ04], [Lie08], [Sta08]

    Content: Present de Jong's proof of the period period–index conjecture for surfaces [dJ04]; see also [Sta08, Chapter 4].

12. **Period–index and Hodge theory** (SP)

    Date: 30.6.

    Reading: [dJP22]

    Content: Discuss the results in [dJP22]

13. **Period–index and further directions** (DM)

    Date: 7.7.

    Reading: [HM25], [HP24], [Hot25]

    Content: Depending on the interest of the speaker, present the main results of (at least one) of the papers mentioned above.

References

[Bea16] A. Beauville, *The Lüroth problem*, in Rationality problems in algebraic geometry, Lecture Notes in Mathematics 2172, Springer, 2016, 1–27.

[CTS21] J.-L. Colliot-Thélène and A. N. Skorobogatov, *The Brauer–Grothendieck group*, Ergebnisse der Mathematik und ihrer Grenzgebiete 71, Springer, 2021.

[dJ] A. J. de Jong, *A result of Gabber*, available at `https://www.math.columbia.edu/~dejong/`.

[dJ04] A. J. de Jong, *The period-index problem for the Brauer group of an algebraic surface*, Duke Math. J. 123 (2004), 71–94.

[dJP22] A. J. de Jong and A. Perry, *The period-index problem and Hodge theory*, Preprint 2022, `https://arxiv.org/abs/2212.12971`

[dJS10] A. J. de Jong and J. Starr, *Almost proper GIT-stacks and discriminant avoidance*, Doc. Math. 15 (2010), 957–972.

[GS06] P. Gille and T. Szamuely, *Central Simple Algebras and Galois Cohomology*, Cambridge Studies in Advanced Mathematics, Cambridge University Press, 2006.

[GH26] F. Gounelas and D. Huybrechts, *Universal Brauer–Severi varieties*, Preprint, 2026, `https://arxiv.org/pdf/2510.20474`.

[HM25] D. Huybrechts and D. Mattei, *Splitting unramified Brauer classes by abelian torsors and the period-index problem*, Math. Ann. 392 (2025), 2913–2932.

[Hot25] J. Hotchkiss, *The period-index problem for complex tori*, IMRN (2025).

[HP24] J. Hotchkiss and A. Perry, *The period-index conjecture for abelian threefolds and Donaldson–Thomas theory*, Preprint, 2024, `https://arxiv.org/abs/2405.03315`.

[Lie08] M. Lieblich, *Twisted sheaves and the period-index problem*, Compos. Math. 144 (2008), 1–31.

[Mil80] J. S. Milne, *Étale cohomology*, Princeton Mathematical Series 33, Princeton University Press, 1980.

[Ols16] M. Olsson, *Algebraic spaces and stacks*, American Mathematical Society Colloquium Publications 62, AMS, 2016.

[Sta08] J. Starr, *Brauer groups and Galois cohomology of function fields of varieties*, IMPA Mathematical Publications, 2008.

[vB24] D. van Bree, *Virasoro constraints and moduli of twisted sheaves*, PhD thesis, Utrecht University, 2024.