

Working Seminar: Resolution of singularities
Thursday 14:15-15:45 in building 1104, room B227

This seminar discusses resolution of singularities and alterations. The first 5 talks are comparatively elementary and discuss various beautiful aspects of resolution of curves and surfaces, as outlined by Kollár in [Kol07, §1-2]. The second half of the seminar discusses a modern treatment of Hironaka's resolution of singularities in characteristic zero, following mostly [Wlo05, Kol07], as well as de Jong's alterations [deJ96] in arbitrary characteristic. The program is provisional and open to discussions. The talks will be distributed during the first meeting on Oct 13. Please contact me (schreieder@math.uni-hannover.de) if you cannot attend the first meeting but would like to give a talk.

1. **Introduction and distribution of talks** (Schreieder)
Date: 13.10.2022
2. **Resolution of curves: Newton, Riemann and Albanese** (Cheng)
Date: 20.10.2022
Reading: [Kol07, §1.1–1.3]
Content: Explain the methods of Newton, Riemann and Albanese.
3. **Normalization and strong embedded resolution of curves** (Nikolov)
Date: 27.10.2022
Reading: [Kol07, §1.4–1.6]
Content: Explain how normalizations resolve curve singularities and how to do it explicitly in examples (cf. [Kol07, 1.31]). Prove [Kol07, Theorems 1.43, 1.47, and 1.52].
4. **Birational transforms of plane curves, local embedded resolution and principalization** (Paulsen)
Date: 3.11.2022
Reading: [Kol07, §1.7-1.9]
Content: Sketch the results in [Kol07, §1.7-1.9].
5. **Resolution of surface singularities I** (Sertöz)
Date: 10.11.2022
Reading: [Kol07, §2.1-2.4]
Content: Sketch the results and examples in [Kol07, §2.1-2.4]. In particular, discuss Ade singularities, Jung's method, and how to resolve quotient singularities. You can assume the basic surface theory facts from [Kol07, §2.2].
6. **Resolution of surface singularities II** (Balkan)
Date: 17.11.2022
Reading: [Kol07, §2.5 and §2.6]
Content: Discuss [Kol07, §2.5-2.6], which gives a complete proof for resolution of surfaces starting with the Albanese method.

7. **Hironaka's proof I** (Pavic)

Date: 1.12.2022
 Reading: [Kol07, §3]
 Content: Explain the main result, examples and problems as outlined in [Kol07, §3.1-3.3].
8. **Hironaka's proof II** (Valloni)

Date: 8.12.2022
 Reading: [Kol07, §3] and [Ann, §4]
 Content: Present the first half of the proof as e.g. outlined in [Ann, §4.1-4.3]. More details are contained in [Kol07, §3].
9. **Hironaka's proof III** (Alexandrou)

Date: 12.1.2023
 Reading: [Kol07, §3] and [Ann, §4]
 Content: Present the second half of the proof as e.g. outlined in [Ann, §4.4-4.8]. More details are contained in [Kol07, §3].
10. **Alterations I** (de Gaay Fortman)

Date: 19.1.2023
 Reading: [deJ96, §1-3]
 Content: Explain the main result of [deJ96], some of the key notions (like alterations) from [deJ96, §2] and [deJ96, §3]. You may also want to mention the generalizations by Gabber and Temkin in [IT14, Tem17].
11. **Alterations II** (Lüders)

Date: 26.1.2023
 Reading: [deJ96, §4]
 Content: Prove that alterations of varieties over fields of arbitrary characteristic exist, see [deJ96, Theorem 4.1].

Literatur

- [Ann] T. Annala, *Resolution of Singularities*, <https://personal.math.ubc.ca/~tannala/blowups.pdf>
- [deJ96] A.J. de Jong, *Smoothness, semi-stability and alterations*, Publ. Math. de l'IHÉS **83** (1996), 51–93.
- [IT14] L. Illusie and M. Temkin, *Exposé X. Gabber's modification theorem (log smooth case)*, Astérisque **363–364** (2014), 167–212, Travaux de Gabber sur l'uniformisation locale et la cohomologie étale des schémas quasi-excellents.
- [Kol07] J. Kollár, *Lectures on Resolution of Singularities*, Princeton University Press, Princeton and Oxford, 2007.
- [Tem17] M. Temkin, *Tame distillation and desingularization by p -alterations*, Ann. Math. **186** (2017), 97–126.
- [Wlo05] J. Włodarczyk, *Simple Hironaka Resolution in Characteristic Zero*, J. Amer. Math. Soc. **18** (2005), 779–822.