Prof. Dr. Stefan Schreieder Sommersemester 2022

Working Seminar: 0-cycles

Thursday 14:15-15:45 in F107

This seminar discusses some of the classical results and conjectures on 0-cycles on surfaces, or more generally on smooth projective varieties. The program is provisional and open to discussions. Please contact me (schreieder@math.uni-hannover.de) if you want to give a talk.

1. **Mumford's theorem** (Paucar/Schreieder)

   Date: 21.4.2022

   Reading: [Voi03, §10]

   Content: Briefly recall what it means that the Chow group of zero-cycles of a complex projective variety is representable, see [Voi03, §10.1]. Sketch the proof of [Voi03, Proposition 10.12] (you may state [Voi03, Lemma 10.13] without proof). Give a proof of [Voi03, Corollary 10.21]; instead of the Hilbert scheme argument used in *loc. cit.*, use the quicker original argument of Bloch–Srinivas in [BS83, Proposition 1]. Use this to prove [Voi03, Theorem 10.17], see [Voi03, §10.2.2], state [Voi03, Corollary 10.18]. Conclude (a weak form of) Mumfords theorem: A smooth complex projective surface S whose Chow group of zero-cycles is representable (or equivalently, finite-dimensional) satisfies $h^{2,0}(S) = 0$, cf. [Voi03, Theorem 10.15].

2. **Bloch's p_g-conjecture** (Ma)

   Date: 28.4.2022

   Reading: [Voi03, §11]

   Content: State Bloch's conjecture [Voi03, Conjecture 11.2]; note that by [Voi03, Theorem 10.11], the conjecture is exactly the converse of (the weak version of) Mumford's theorem proven last time. Prove Bloch's conjecture for surfaces not of general type, see [Voi03, §11.1.2 and §11.1.3]. Sketch the proof of Bloch's conjecture for Godeaux surfaces, see [Voi03, §11.1.4].

3. **Generalized Bloch conjecture** (Mezzedimi)

   Date: 12.5.2022

   Reading: [HK13, Voi12, Huy12]

   Content: State the generalized Bloch conjecture, e.g. in the form [Voi03, Conjecture 11.19] (see also [Voi12, Conjecture 0.1 and 0.2]). Deduce from this conjecture that a symplectic automorphism of a K3 surface should act trivially on the Chow group of zero cycles, see [HK13, Conjecture 0.1]. Prove this for automorphisms of order 2, following [Voi03]. State the more general result in [Huy12] and sketch its proof if time permits.

4. **Green's conjecture** (Floccari)

   Date: 2.6.2022

   Reading: [Voi99, Gre98]

   Content: Explain Green's conjecture on higher Abel–Jacobi mappings, which would imply Bloch's p_g-conjecture. Show that Green's original candidate for a higher Abel–Jacobi map does not have the required property, see [Voi99]. If time permits, explain that Green's map is however strong enough to prove Mumford's theorem, see [Voi99].

5. **Cycle class maps in continuous ℓ-adic étale cohomology and Jannsen's injectivity conjecture** (Balkan/Schreieder)

   Date: 16.6.2022

   Reading: [Jan88, Jan94]

   Content: Sketch the construction of continuous étale cohomology and of cycle classes in that theory, see [Jan88]. Explain the result mentioned in [Jan88, Remark 6.15(a)], cf. [Sch22, Step 8 in Proposition 6.6]. State Jannsen's conjecture on the injectivity of his cycle class map over finitely generated fields, see e.g. [Jan94, Question 2.8]. Show that a positive answer to Question 2.8 would imply the Bloch–Beilinson conjecture and hence Bloch's p_g-conjecture, see [Jan88, Lemma 2.7]. (You can use a limit argument to reduce the case of general fields to that of finitely generated fields.)

6. **Constant cycle curves** (Sertöz)

   Date: 23.6.2022

   Reading: [Huy14]

   Content: Explain some of the results in [Huy14]. In particular, give the various definitions of constant cycle curves and their order, see [Huy14, Definition 3.1, 3.2, and 3.5]. Compare the two definitions, see [Huy14, Proposition 3.7]. Explain the relation to Bloch's conjecture, see [Huy14, Proposition 4.1]. Give various examples of constant cycle curves on K3 surfaces, see [Huy14, §6–7] and Voisin's construction in [Huy14, Theorem A.1]. If time permits, sketch the proof of [Huy14, Proposition 5.1].

7. **Roitman's theorem** (Alexandrou)

   Date: 7.7.2022

   Reading: [Blo79, Roi80]

   Content. Give a sketch of the proof of Roitman's theorem, either following the original approach of Roitman [Roi80] or the algebraic approach of Bloch [Blo79].

8. **Torsion codimension 2 cycles over number fields** (Valloni)

   Date: 14.7.2022

   Reading: [MS83, Sai91]

   Content: Mention the result of Merkujev–Suslin: torsion codimension 2-cycles over an algebraically closed field of characteristic zero inject via the Abel –Jacobi map into the respect intermediate Jacobian, see [MS83, §18]. Explain that this generalizes Roitman's theorem from the previous meeting, as the latter reduces by the Lefschetz hyperplane theorem to the case of surfaces.

   The main content of the talk is then the study of torsion codimension 2-cycles over non-closed fields. In particular, explain some of the results on the injectivity of the (integral) cycle class map on torsion codimension 2 cycles from [Sai91], which generalizes previous results of Colliot-Thélène and Raskind in [CTR91].

9. **Zero-cycles over p-adic fields** (Lüders)

   Date: 21.7.2022

   Reading: [SS10]

   Content: Present the main results from [SS10].

Literatur

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[BS83] S. Bloch and V. Srinivas, Remarks on correspondences and algebraic cycles, Amer. J. Math. **105** (1983), 1235–1253.

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[Gre98] M. Green, *Higher Abel-Jacobi maps*, Proceedings of the International Congress of Mathematicians, Vol. II (Berlin, 1998). Doc. Math. 1998, Extra Vol. II, 267–276.

[HK13] D. Huybrechts and M. Kemeny, *Stable maps and Chow groups*. Doc. Math. **18** (2013), 507–517.

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[Jan94] U. Jannsen, *Motivic sheaves and filtrations on Chow groups*, Proceedings of Symposia in Pure Mathematics **55** (1994), 245–302.

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[Voi99] C. Voisin, *Some results on Green's higher Abel-Jacobi map*, Ann. of Math. **149** (1999), 451–473.

[Voi03] C. Voisin, *Hodge Theory and Complex Algebraic Geometry II*, Cambridge University Press, Cambridge 2003.

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