Let $X$ be a smooth projective variety of dimension $d$. An object $S$ in $\mathcal{D}^b(X)$ is called spherical if

1. $\text{Ext}^\bullet(S, S) = \mathbb{k} \oplus \mathbb{k}[-d]$; $d$-spherelike object
2. $S \otimes \omega_X \cong S$. Calabi-Yau object

P. Seidel and R. Thomas showed that such an $S$ defines an autoequivalence $T_S$, the spherical twist about $S$. In the context of the homological mirror symmetry conjecture, they proved that $T_S$ is dual to the Dehn twist of a symplectic manifold about a Lagrangian.

In general, the Calabi-Yau property of a spherical object will be lost under birational transformations of $X$. In a joint work, M. Kalck, D. Ploog and I showed how to associate to an arbitrary spherelike object $F$ in a triangulated category $\mathcal{D}$ a unique maximal triangulated subcategory $\mathcal{D}_F$, where $F$ becomes spherical — the spherical subcategory of $F$.

In this talk, I will give a short introduction to spherical subcategories and then will focus on the case of spherelike divisors $D$ on a surface $X$, i.e. effective divisors such that $\mathcal{O}_D$ is spherelike. Especially, I will talk about a numerical characterisation of these divisors and to what extent they can be classified.

This is work in progress with D. Ploog.