OBERSEMINAR ALGEBRAISCHE GEOMETRIE

LOGARITHMIC CHERN SLOPES OF LOG-SURFACES

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In the 1970s, Japanese School of Algebraic Surfaces introduced an interesting topic of studies, namely log-surfaces. Let us recall very briefly the main idea standing behind this subject. Consider a pair (X, D), where X is a smooth projective surface and D a simple normal crossing divisor, then we can compute the Chern numbers of this pair:

$$\overline{c}_1^2(X,D) = c_1(\Omega_X^1(\log(D))^\vee)^2,$$
$$\overline{c}_2(X,D) = c_2(\Omega_X^1(\log(D))^\vee).$$

Over the complex numbers and under the assumption that the log-Kodaira dimension of (X,D) is non-negative, it is well-known that the following variation of the Bogomolov-Miyaoka-Yau inequality holds (this is a result due to F. Sakai from 1980):

$$\overline{c}_1^2(X,D) \leq 3\overline{c}_2(X,D).$$

Now we consider a reduced divisor $\mathcal{C} \subset \mathbb{P}^2_{\mathbb{C}}$, which is a configuration of $k \geq 3$ curves, satisfying the following properties:

- all irreducible components are smooth of degree $d \geq 1$,
- there is no point where all curves meet,
- all intersection points are *ordinary*, i.e., locally they look like intersections of lines.

We blow up $\mathcal{C}$ along points having multiplicities greater or equal to 3, and we define D to be reduced total transform of $\mathcal{C}$. The above considerations lead us to the pair (X, D), where X is the blowing-up of $\mathbb{P}^2_{\mathbb{C}}$. In 1984, A. J. Sommese proved that if D is a line arrangement in $\mathbb{P}^2_{\mathbb{C}}$, then

$$\overline{c}_1^2(X,D) \leq \frac{8}{3}\overline{c}_2(X,D),$$

and the equality holds iff D is the dual Hesse arrangement of lines.

In the talk, we will generalize Sommese's result to curve configurations $\mathcal{C}$ in $\mathbb{P}^2_{\mathbb{C}}$ of degree ≥ 2. Our prove uses heavily Hirzebruch's construction of abelian covers branched along curve configurations and combinatorics. I will present an interesting configuration of conics in $\mathbb{P}^2_{\mathbb{C}}$ which is constructed via polars of Klein's arrangement of lines. This configuration consists of 21 conics with 224 triple and 168 double points as the intersections and, which is the crucial thing, it allows us to produce the largest-known logarithmic Chern slop in the class of conics arrangements equal to $c_1^2/c_2 \approx 2.25$.